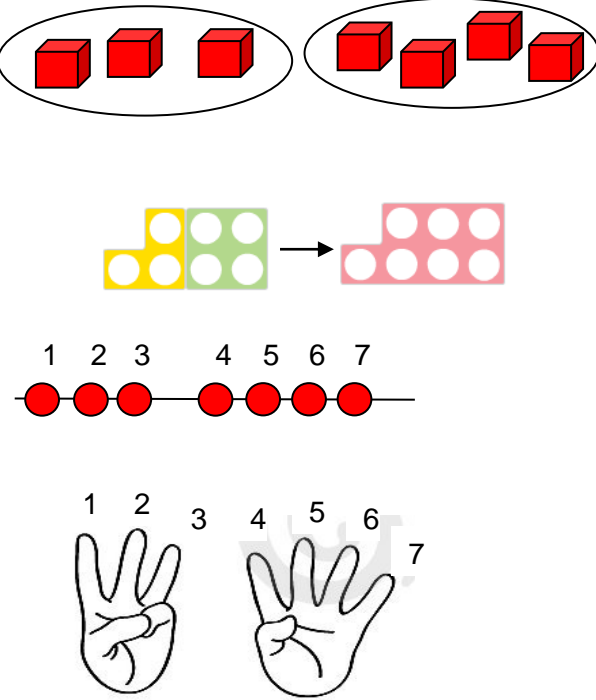
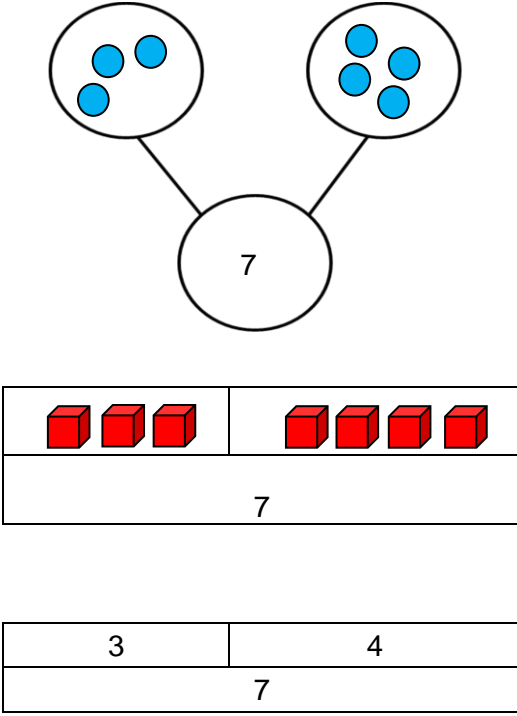


Calculation: Addition

Key language: add, plus, total, altogether, more, sum, part, whole, equal to, column, partition, value, worth, ones, tens, hundreds

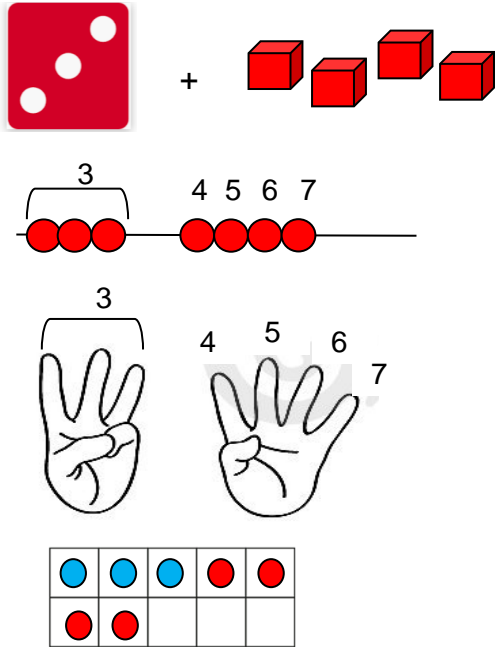
Key representations: Base 10, bead strings, multilink, counters, Numicon, bar model, number lines, ten frames, fingers, place value counters, 100 square

Concrete	Pictorial	Abstract	Examples of Stem Sentences
<p>Aggregation-count all Adding groups to make a whole:</p> <p style="text-align: center;">$3 + 4 = 7$</p> 	<p style="text-align: center;">$3 + 4 = 7$</p> 	<p style="text-align: center;">$3 + 4 = 7$</p>	<p>'__ plus __ is equal to __'</p> <p>'__ is equal to __ plus __'</p> <p>'__ is the sum of __ and __'</p>

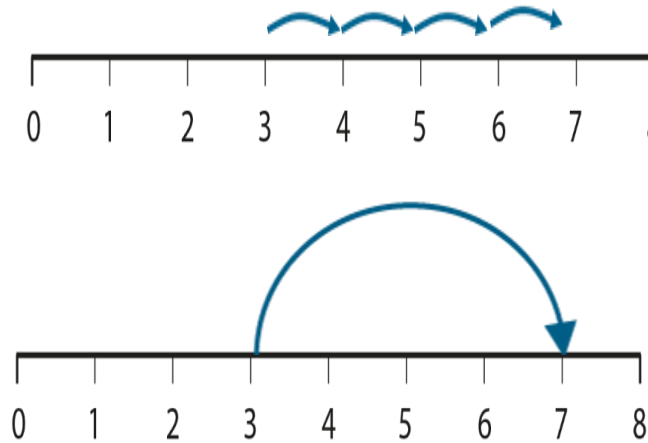
NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-105-additive-structures-introduction-to-aggregation-and-partitioning/>

Augmentation-count on
Count on for addition:

$$3 + 4 = 7$$



$$3 + 4 = 7$$



$$3 + 4 = 7$$

'First... then... now'

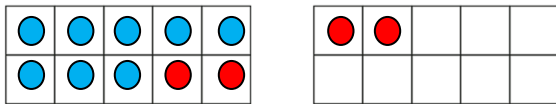
'First Jack has 3 sweets, then he was given 4 more, now he has 7 sweets'

NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-106-additive-structures-introduction-to-augmentation-and-reduction/>

Bridging through 10:

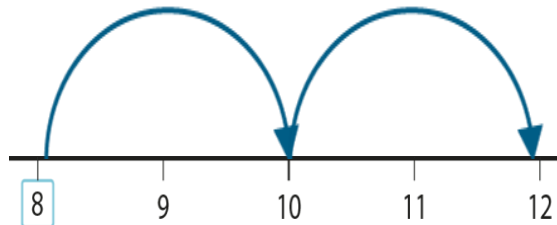
$$8 + 4 =$$

$$8 + 2 + 2 =$$



$$8 + 4 =$$

$$8 + 2 + 2 =$$



$$8 + 4 =$$

$$8 + 2 + 2 =$$

$$8 + 4 =$$

$$\begin{array}{l} \swarrow \searrow \\ 2 \quad 2 \end{array}$$

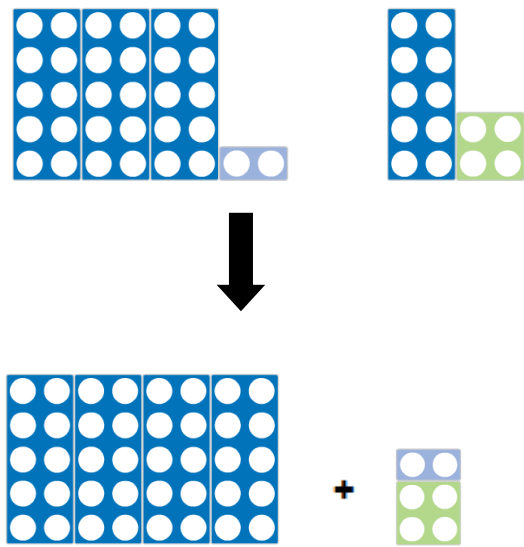
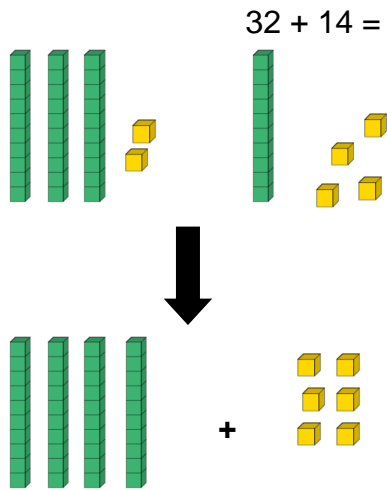
'First I partition the 4-2 plus 2 equals 4.

Then 8 plus 2 is equal to 10.

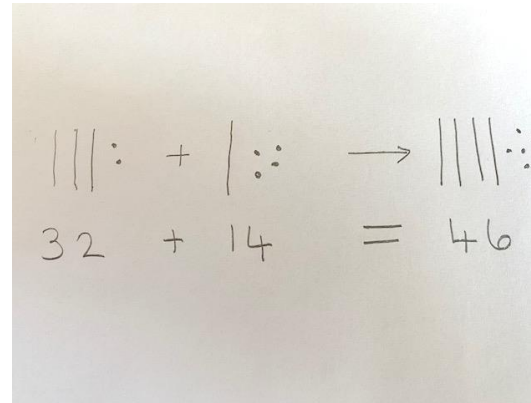
10 plus 2 is equal to 12'

NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-111-addition-and-subtraction-bridging-10/>

Partitioning and Recombining- to support mental addition



$$32 + 14 =$$



$$32 + 14 =$$

$$30 + 10 = 40$$

$$2 + 4 = 6$$

$$40 + 6 = 46$$

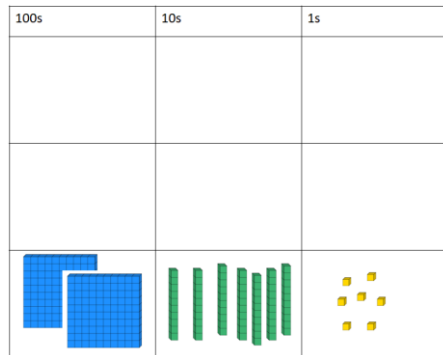
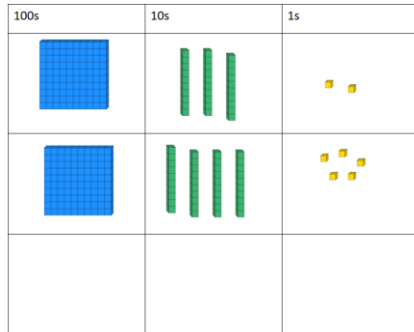
'First, I add the tens. 3 tens and 1 ten is equal to 4 tens.'

Then I add the ones. 2 ones and 4 ones is equal to 6 ones.'

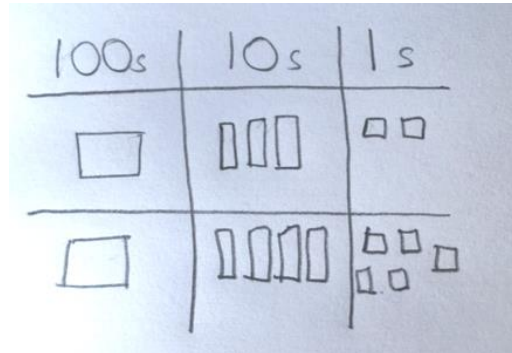
4 tens and 6 ones is equal to 46.'

Column Addition-no exchange:

$$\begin{array}{r} 132 \\ +145 \\ \hline \end{array}$$



$$\begin{array}{r} 132 \\ +145 \\ \hline \end{array}$$



132	145
?	

$$\begin{array}{r} 132 \\ +145 \\ \hline 277 \end{array}$$

'We line up the ones: 2 ones plus 5 ones equals 7 ones.'

'We line up the tens: 3 tens and 4 tens equals 7 tens.'

'We line up the hundreds: 1 hundred plus 1 hundred equals 2 hundreds.'

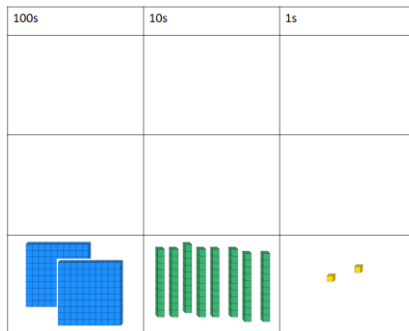
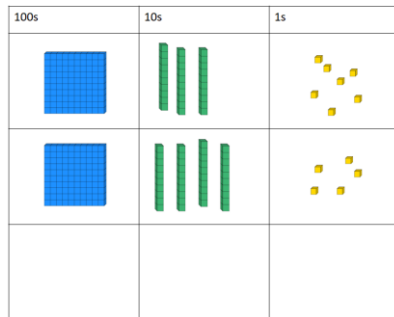
'2 ones plus 5 ones is equal to 7 ones.'

'3 tens plus 4 tens is equal to 7 tens.'

'1 hundred plus 1 hundred is equal to 2 hundreds.'

Column Addition with exchange:

$$\begin{array}{r} 137 \\ +145 \\ \hline 282 \\ \hline \end{array}$$



$$\begin{array}{r} 137 \\ +145 \\ \hline 282 \\ \hline \end{array}$$

137	145
?	

$$\begin{array}{r} 137 \\ +145 \\ \hline 282 \\ \hline \end{array}$$

'If the column sum is equal to 10 or more, we must regroup.'

'7 ones plus 5 ones is equal to 12 ones.'

I regroup 12 ones into 1 ten and 2 ones and place the 10 in the tens column.

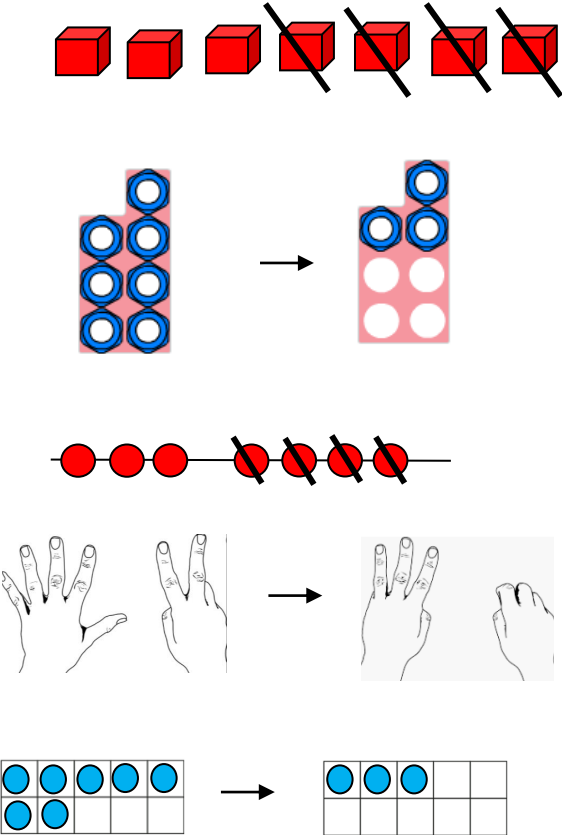
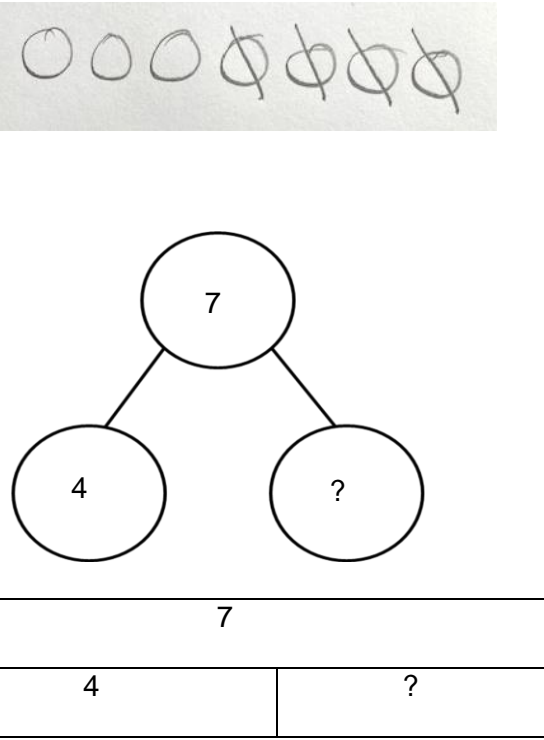
3 tens plus 4 tens plus 1 ten is equal to 8 tens.

1 hundred plus 1 hundred is equal to 2 hundreds.'

Calculation: Subtraction

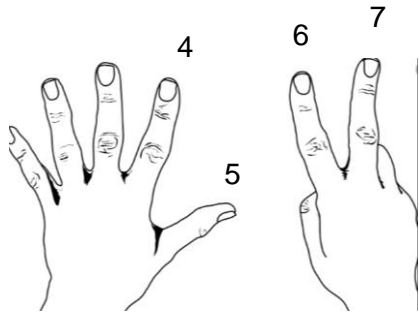
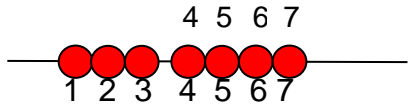
Key language: minus, subtract, take away, left, less than, difference, fewer, decrease, part, whole, equal to, column, partition, value, worth, ones, tens, hundreds

Key representations: Base 10, bead strings, multilink, counters, Numicon, bar model, number lines, ten frames, fingers, place value counters, 100 square

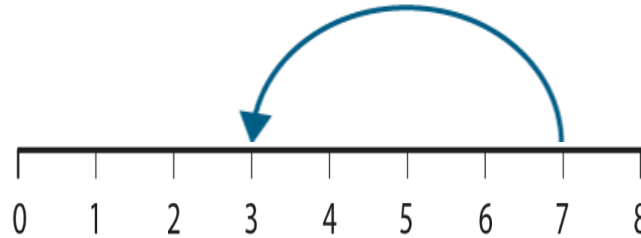
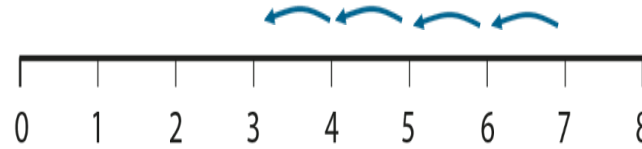
Concrete	Pictorial	Abstract	Stem Sentences
<p>Reduction: Take away $7 - 4 = 3$</p> 	<p>$7 - 4 = 3$</p> 	<p>$7 - 4 = 3$</p>	<p><i>'First... then... now'</i></p> <p><i>'First Jack had 7 sweets. Then he ate 4. Now he has 3 sweets left. 7 minus 4 equals 3.'</i></p>
<p>NCETM Mastery Professional Development Materials: https://www.ncetm.org.uk/classroom-resources/primm-106-additive-structures-introduction-to-augmentation-and-reduction/</p>			

Count back

$$7 - 4 = 3$$



$$7 - 4 = 3$$

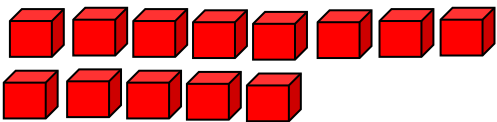


$$7 - 4 = 3$$

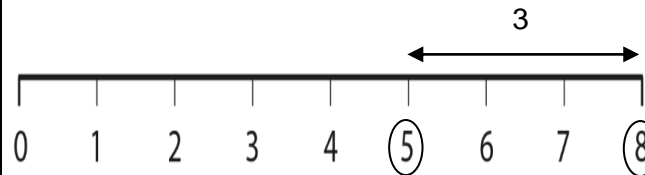
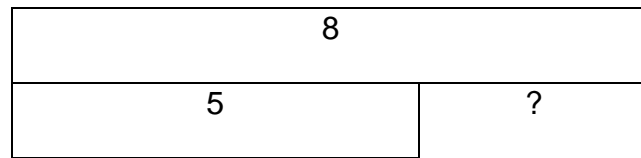
'Count back 4 from 7.'

Comparison: Finding the difference

$$8 - 5 = 3$$



$$8 - 5 = 3$$



$$8 - 5 = 3$$

'The difference between 8 and 5 is 3.'

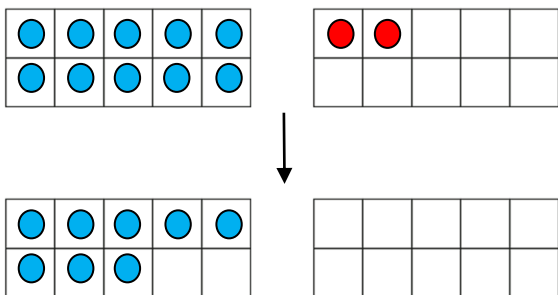
'The difference between 5 and 8 is 3.'

'3 is the difference between 8 and 5.'

Bridging through 10-to support mental subtraction

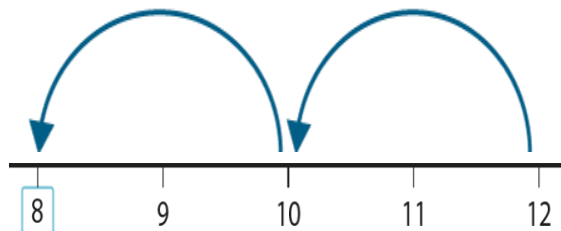
$$12 - 4 =$$

$$12 - 2 - 2 =$$



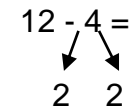
$$12 - 4 =$$

$$12 - 2 - 2 =$$



$$12 - 4 =$$

$$12 - 2 - 2 =$$



'First I use my knowledge of place value to decide what needs to be subtracted from 12 to make a multiple of 10.'

I partition 4 into 2 and 2.

Then 12 minus 2 is equal to 10.

I use my knowledge of number bonds to 10 to calculate 10 minus 2.

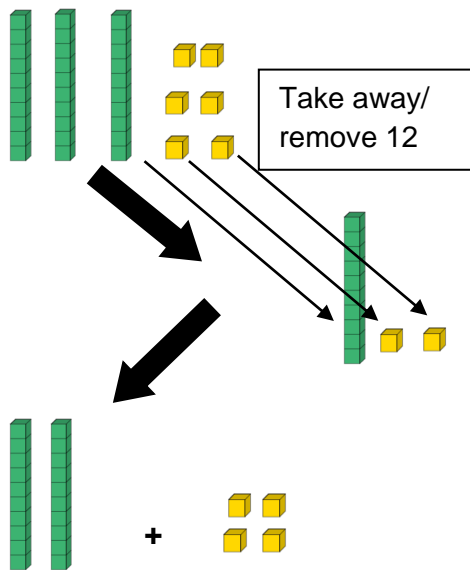
10 minus 2 is equal to 8.

So 12 minus 4 is equal to 8.'

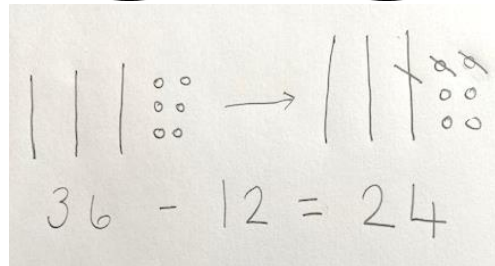
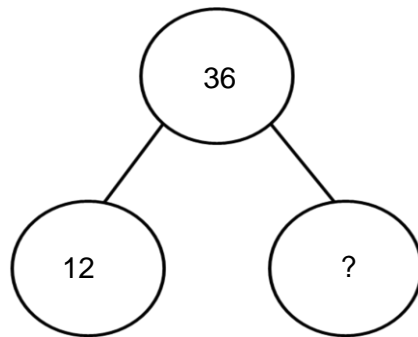
NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-111-addition-and-subtraction-bridging-10/>

Partitioning and Recombining- to support mental subtraction

$$36 - 12 =$$



$$36 - 12 =$$



$$36 - 12 =$$

$$30 - 10 = 20$$

$$6 - 2 = 4$$

$$20 + 4 = 24$$

'First, I subtract the tens. 3 tens minus 1 ten is equal to 2 tens.'

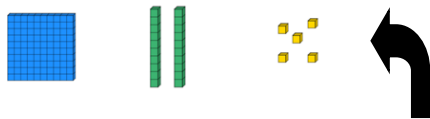
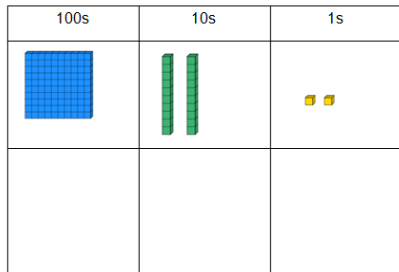
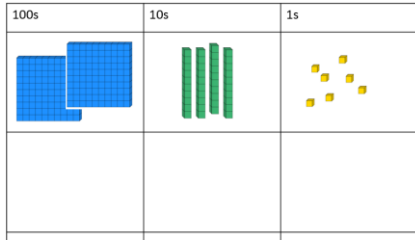
Then I subtract the ones. 6 ones minus 2 ones is equal to 4 ones.

2 tens and 4 ones is equal to 24.'

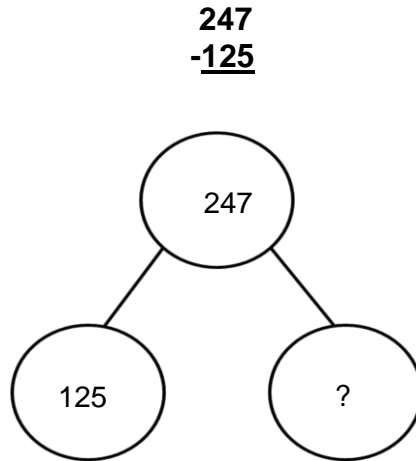
NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-116-subtraction-two-digit-and-two-digit-numbers/>

Column Subtraction-no exchange:

$$\begin{array}{r} 247 \\ - 125 \\ \hline \end{array}$$



Take away/
remove 125



247	
125	?

$$\begin{array}{r} 247 \\ -125 \\ \hline 122 \end{array}$$

'We line up the ones: 7 ones minus 5 ones equals 2 ones.'

'We line up the tens: 4 tens minus 2 tens equals 2 tens.'

'We line up the hundreds: 2 hundreds minus 1 hundred equals 1 hundred.'

'7 ones minus 5 ones is equal to 2 ones.'

4 tens minus 2 tens is equal to 2 tens.'

2 hundreds minus 1 hundred is equal to 1 hundred.'

Column Subtraction with exchange:

$$\begin{array}{r} 235 \\ - 118 \\ \hline \end{array}$$

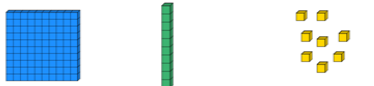
100s	10s	1s



100s	10s	1s

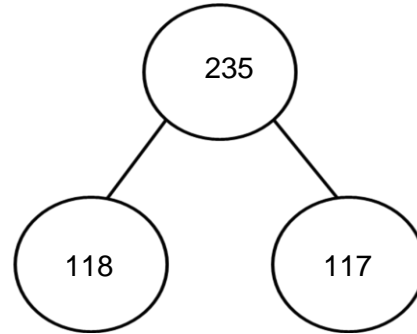


100s	10s	1s



Take away/
remove 118

$$\begin{array}{r} 235 \\ - 118 \\ \hline \end{array}$$



235	
118	?

$$\begin{array}{r} 235 \\ - 118 \\ \hline 117 \end{array}$$

'5 ones minus 8 ones...we need to exchange 1 ten for 10 ones.

15 ones minus 8 ones is equal to 7 ones.

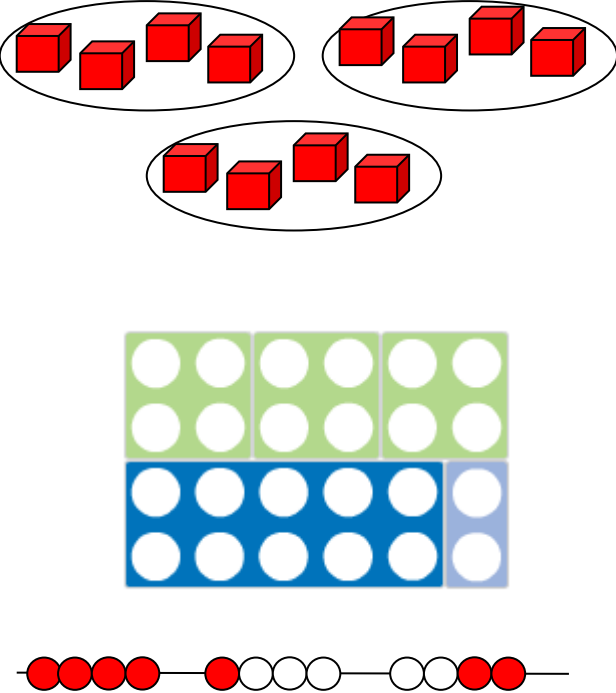
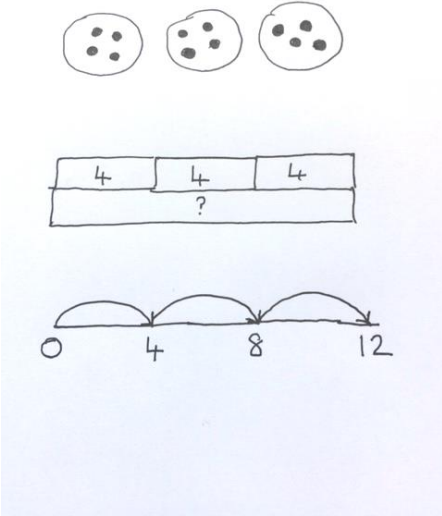
2 tens minus 1 ten is equal to 1 ten.

2 hundreds minus 1 hundred is equal to 1 hundred.'

Calculation: Multiplication

Key language: double, times, multiplied by, product, groups of, lots of, equal groups, arrays, factor, multiple, partition, commutative, column, value, multiple

Key representations: place value counters, base 10, bead strings, multilink, counters, Numicon, bar model, number line

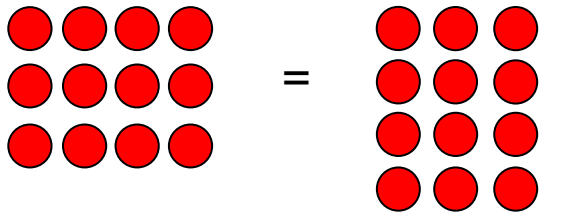
Concrete	Pictorial	Abstract	Stem Sentences
<p>Equal groups:</p> $3 \times 4 = 12$ $4 + 4 + 4 = 12$ 	$3 \times 4 = 12$ $4 + 4 + 4 = 12$ 	$3 \times 4 = 12$ $4 + 4 + 4 = 12$	<p><i>'There are __ equal groups.'</i></p> <p><i>There are __ in each group.'</i></p> <p><i>There are __ equal groups of __.'</i></p> <p><i>'If there are __ equal groups, we can use the __ times table.'</i></p>

NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-202-structures-multiplication-representing-equal-groups/>

Arrays to show commutativity:

$$3 \times 4 = 4 \times 3$$

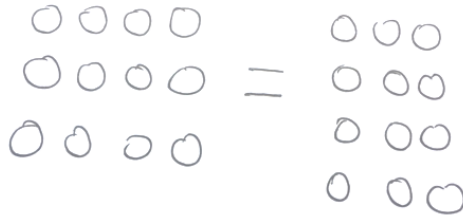
$$4 + 4 + 4 = 3 + 3 + 3 + 3$$



3 groups of 4 = 4 groups of 3

$$3 \times 4 = 4 \times 3$$

$$4 + 4 + 4 = 3 + 3 + 3 + 3$$



$$3 \times 4 = 4 \times 3$$

$$4 + 4 + 4 = 3 + 3 + 3 + 3$$

'I can see 3 groups of 4 and I can see 4 groups of 3.'

'3 times 4 can represent 3 groups of 4.'

It can also represent 4 times 3.'

'If there are ___ equal groups, we can use the ___ times table.'

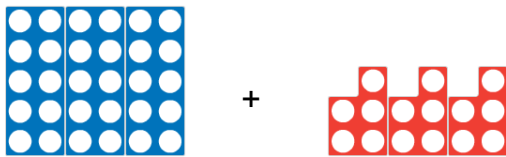
NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-205-commutativity-part-2-doubling-and-halving/>

Partition to multiply:

$$3 \times 15 =$$



becomes



3 x 10 + 3 x 5

$$3 \times 15 =$$

15	15	15
?		



$$3 \times 15 =$$

$$3 \times 10 = 30$$

$$3 \times 5 = 15$$

$$30 + 15 = 45$$

x	10	5
3	30	15

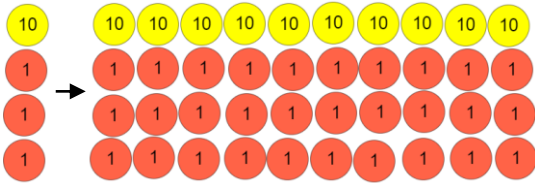
'15 is equal to 10 plus 5.'



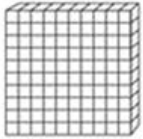
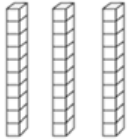
So 3 times 15 is equal to 3 times 10 plus 3 times 5.'

NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-214-multiplication-partitioning-leading-to-short-multiplication/>

Multiply by 10:

$13 \times 10 =$



100s	10s	1s
		
		

$13 \times 10 =$

100s	10s	1s
	1	3
1	3	0

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

13	13	13	13	13	13	13	13	13	13
?									

$13 \times 10 = 130$

'__ multiplied by 10 is equal to __.'

'__ is ten times the size of __.'

Short multiplication: 2 digit by 1 digit with no exchange

$$23 \times 3 =$$



$$60 + 9$$

$$23 \times 3 =$$

23	23	23
?		

$$23 \times 3 =$$

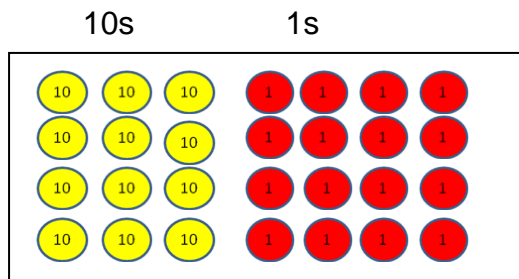
$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

'First we multiply the ones and then we multiply the tens. We add those products together.'

NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-214-multiplication-partitioning-leading-to-short-multiplication/>

Short multiplication: 2 digit by 1 digit with exchange

$$34 \times 4 =$$



$$120 + 16$$

$$34 \times 4 =$$

34	34	34	34
?			

$$34 \times 4 =$$

$$\begin{array}{r} 34 \\ \times 4 \\ \hline 16 \\ +120 \\ \hline 136 \\ \downarrow \\ 34 \\ \times 4 \\ \hline 136 \\ \hline 1 \end{array}$$

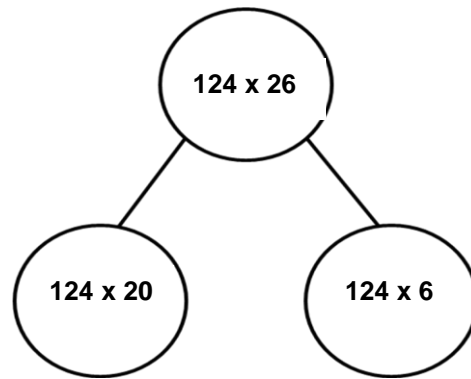
'First we multiply the ones and then we multiply the tens. We add those products together.'

'If there are 10 or more ones, we must regroup the ones into 10s and 1s.'

NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-214-multiplication-partitioning-leading-to-short-multiplication/>

Long multiplication: 3 digit by 2 digit

$$124 \times 26 =$$



$$124 \times 26 =$$

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \end{array}$$

'If there are 10 or more ones, we must regroup the ones into 10s and 1s.'

'If there are 10 or more hundreds, we must regroup then tens into 100s and 10s.'

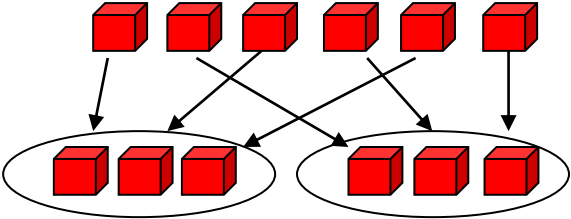
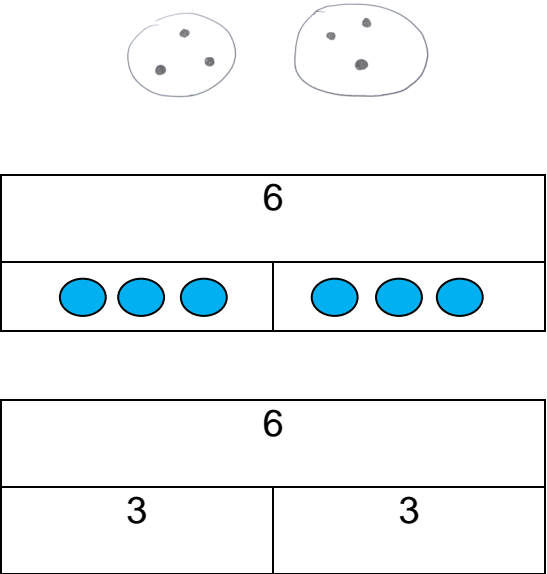
'If there are 10 or more hundreds, we must regroup the hundreds into 1000s and 100s.'

'To multiply a three digit number by a two digit number, first multiply by the ones, then multiply by the tens and then add them together.'

Calculation: Division

Key language: share, group, divide, divided by, half, equal, dividend, divisor, quotient, factor, multiple, remainder

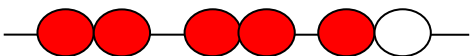
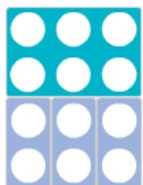
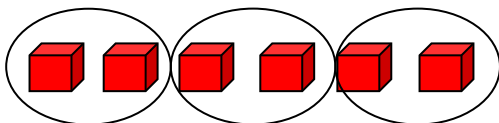
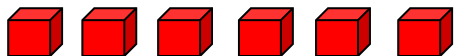
Key representations: place value counters, base 10, bead strings, multilink, counters, Numicon, bar model, number line

Concrete	Pictorial	Abstract	Stem Sentences
<p>Sharing-Partitive: using a range of discrete concrete objects</p> <p>$6 \div 2 = 3$</p> 	<p>$6 \div 2 = 3$</p> 	<p>$6 \div 2 = 3$</p>	<p><i>'6 divided between 2 is equal to 3 each.'</i></p> <p><i>'6 shared into 2 equal groups, there are 3 in each group.'</i></p> <p><i>'6 is the dividend 2 is the divisor 3 is the quotient.'</i></p>

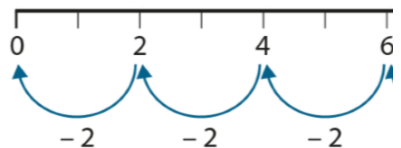
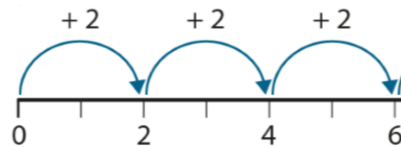
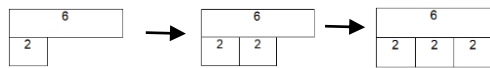
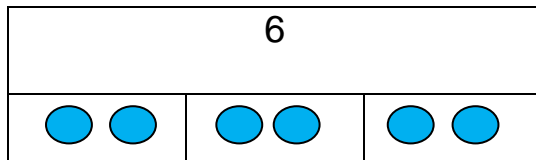
NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-206-structures-quotitive-and-partitive-division/>

Grouping-Quotitive: using a range of concrete objects

$$6 \div 2 = 3$$



$$6 \div 2 = 3$$



$$6 \div 2 = 3$$

'6 is divided into groups of 2. There are 3 groups.'

'6 divided into groups of 2 is equal to 3.'

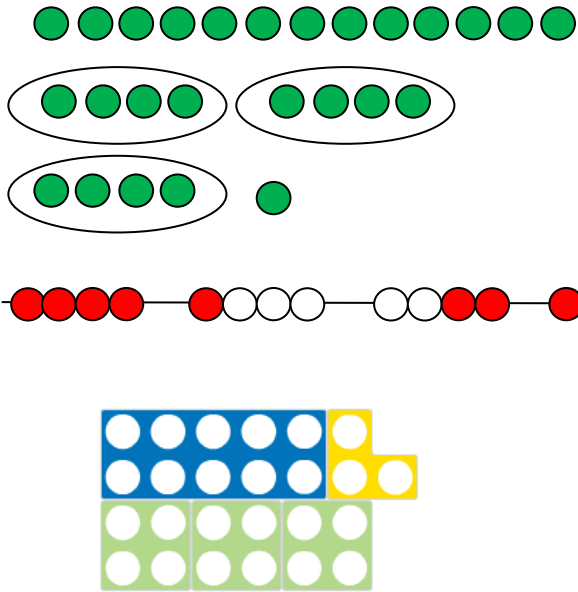
'There are 3 groups of 2 in 6'

*'6 is the dividend
2 is the divisor
3 is the quotient.'*

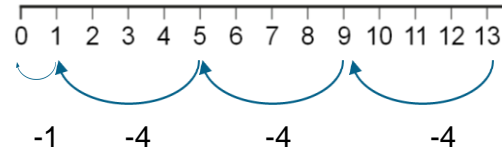
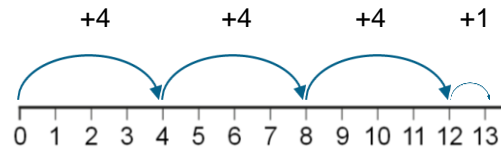
'We can skip count using the divisor to find the quotient.'

Grouping-Quotitive: with remainders

$$13 \div 4 =$$



$$13 \div 4 =$$



$$13 \div 4 = 3 \text{ r } 1$$

'3 is divided into groups of 4. There are 3 groups and a remainder of 1.'

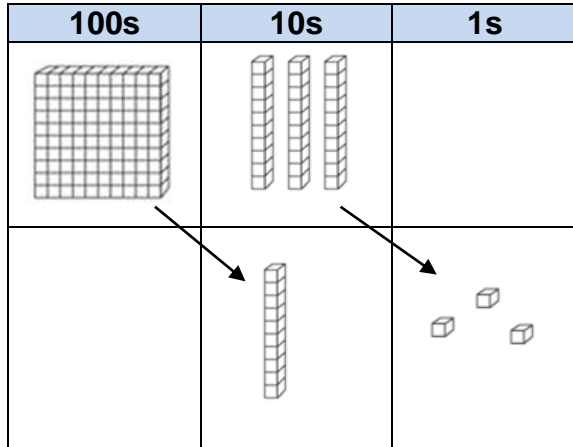
'13 divided into groups of 4 is equal to 3 remainder 1.'

'The remainder is always less than the divisor.'

'If the dividend is not a multiple of the divisor, there is a remainder.'

Divide by 10:

$$130 \div 10 =$$



$$130 \div 10 =$$

100s	10s	1s
1	3	0
	1	3

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
↓ 100	200	300	400	500	600	700	800	900
10	20	↓ 30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

130									
?	?	?	?	?	?	?	?	?	?

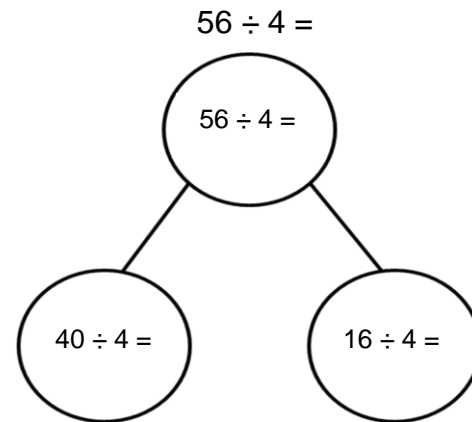
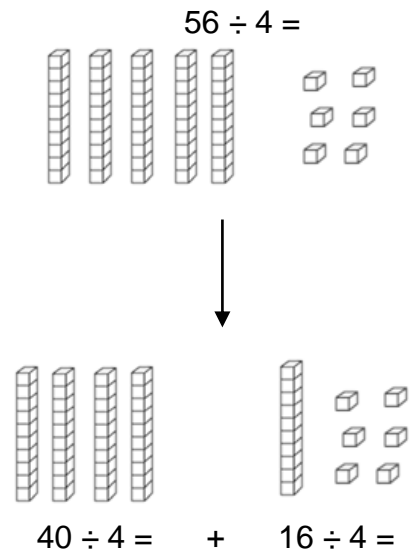
$$130 \div 10 = 13$$

'__ divided by 10 is equal to __.'

'__ is ten times smaller than __.'

Partitioning: to support mental calculation

(Only use when the dividend is beyond the twelfth multiple. Before the twelfth multiple, times tables facts should be used).



$$56 \div 4 =$$

Partitive:

'56 is partitioned into 40 and 16.'

'40 divided between 4 is equal to 10 each.'

'16 divided between 4 is equal to 4 each.'

'56 divided between 4 is equal to 1 ten and 4 ones.'

Quotitive:

'56 is partitioned into 40 and 16.'

'40 is divided into groups of 4. There are 10 groups.'

'16 divided into groups of 4 is equal to 4.'

'There are 14 groups of 4 in 56'

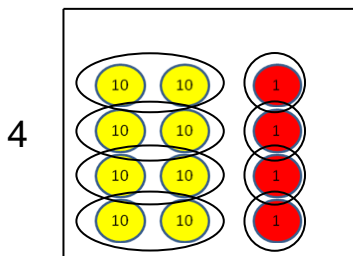
Short division: partitive: 2 digit by 1 digit (with no exchange)

(Only use when the dividend is beyond the twelfth multiple. Before the twelfth multiple, times tables facts should be used).

$$84 \div 4 = \quad 4 \overline{)84}$$

10s 1s

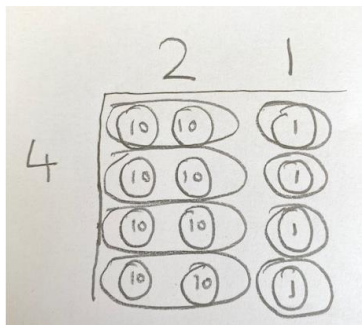
2 1



$$84 \div 4 =$$

$$4 \overline{)84}$$

10s 1s



84			
?	?	?	?

$$84 \div 4 = 21$$

$$\begin{array}{r} 10s \quad 1s \\ 2 \quad 1 \\ 4 \overline{)84} \end{array}$$

'8 tens divided between 4 is equal to 2 tens each.

4 ones divided between 4 is equal to 1 one each.

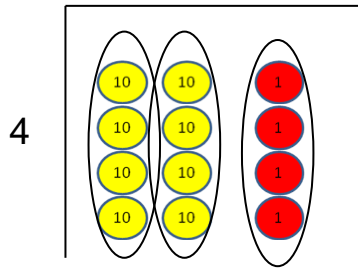
8 tens and 4 ones divided between 4 is equal to 2 tens and 1 one, which is 21.

Short division: quotitive 2 digit by 1 digit (with no exchange)

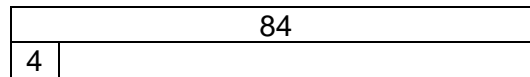
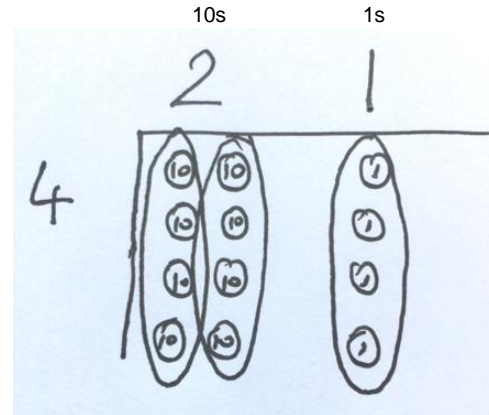
(Only use when the dividend is beyond the twelfth multiple. Before the twelfth multiple, times tables facts should be used).

$$84 \div 4 = \quad 4 \overline{)84}$$

10s	1s
2	1



$$84 \div 4 = \quad 4 \overline{)84}$$



$$84 \div 4 = 21$$

$$\begin{array}{r} 10s \quad 1s \\ 2 \quad 1 \\ 4 \overline{)84} \end{array}$$

'8 tens are divided into groups of 4. There are 2 groups.'

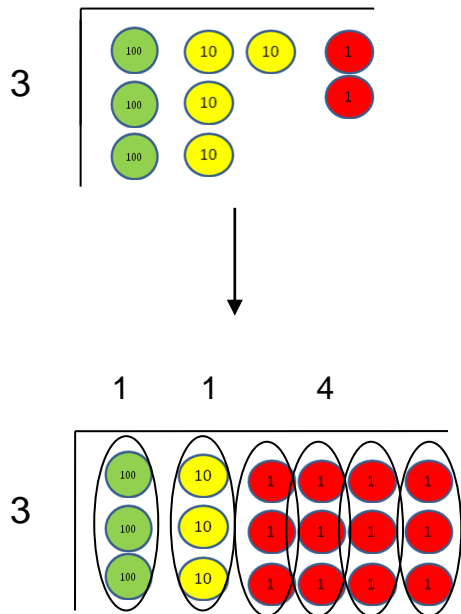
'4 ones are divided into groups of 4. There is 1 group.'

2 groups of ten and 1 one is 21. The quotient is 21.

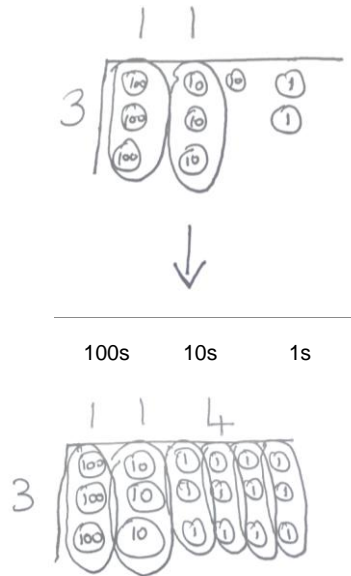
Short division: quotitive 2 digit by 1 digit (with exchange)

(Only use when the dividend is beyond the twelfth multiple. Before the twelfth multiple, times tables facts should be used).

$$342 \div 3 = 3 \overline{)342}$$



$$342 \div 3 = 3 \overline{)342}$$



$$342 \div 3 = 114$$

$$\begin{array}{r} 100s \quad 10s \quad 1s \\ 1 \quad 1 \quad 4 \\ 3 \overline{)342} \end{array}$$

'3 hundreds divided by 3 is 1 hundred.

4 tens divided by 3 is 1 group remainder 1 ten.

Exchange 1 ten for 10 ones.

12 ones divided by 3 is equal to 4.'

Formal division: with 2-digit divisors

Short Division

$$465 \div 31 =$$

$$\begin{array}{r} 0 \ 1 \ 5 \\ 31 \overline{) 4 \ 6 \ 5} \\ \underline{3 \ 1} \\ 1 \ 5 \ 5 \\ \underline{1 \ 5} \\ 0 \end{array}$$

(It may be helpful to write a list of multiples of 31 e.g. 31, 62, 93, 124, 155)

Long Division

$$465 \div 31 =$$

$$\begin{array}{r} 0 \ 1 \ 5 \\ 31 \overline{) 4 \ 6 \ 5} \\ \underline{3 \ 1} \\ 1 \ 5 \ 5 \\ \underline{1 \ 5} \\ 0 \end{array}$$

NCETM Mastery Professional Development Materials: <https://www.ncetm.org.uk/classroom-resources/primm-224-division-dividing-by-two-digit-divisors/>